

Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 1 (Individual)

香港數學競賽 (2005 – 2006)

決賽項目 1 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

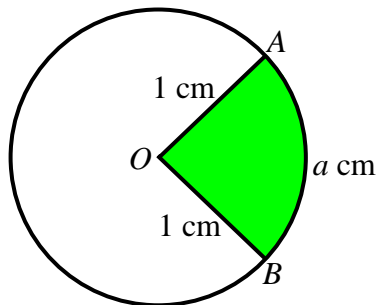
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 a 為實數且滿足方程 $\log_2(x+3) - \log_2(x+1) = 1$ ，求 a 的值。

If a is a real number satisfying the equation $\log_2(x+3) - \log_2(x+1) = 1$, find the value of a .

2. 如圖一， O 是半徑為 1 cm 的圓的圓心。若弧 AB 的長度是 a cm 及著色部分扇形 OAB 的面積是 b cm^2 ，求 b 的值。(取 $\pi = 3$)

In Figure 1, O is the center of the circle with radius 1 cm. If the length of the arc AB is equal to a cm and the area of the shaded sector OAB is equal to b cm^2 , find the value of b . (Take $\pi = 3$)



3. 一個正 C 邊形的一隻內角是 $288b^\circ$ ，求 C 的值。

An interior angle of a regular C -sided polygon is $288b^\circ$, find the value of C .

4. 已知 C 是方程 $kx^2 + 2x + 5 = 0$ 的一個根，其中 k 為常數。若 D 是另一個根，求 D 的值。

Given that C is a root of the equation $kx^2 + 2x + 5 = 0$, where k is a constant. If D is another root, find the value of D .

Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 2 (Individual)

香港數學競賽 (2005 – 2006)

決賽項目 2 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 a 、 b 和 c 三數均不為 0 且 $a:b:c=6:3:1$ 。若 $R = \frac{3b^2}{2a^2 + bc}$ ，求 R 的值。

Given that a , b and c are three numbers not equal to 0 and $a:b:c=6:3:1$. If $R = \frac{3b^2}{2a^2 + bc}$, find the value of R .

2. 已知 $\frac{|k+R|}{|R|} = 0$ 。若 $S = \frac{|k+2R|}{|2k+R|}$ ，求 S 的值。

Given $\frac{|k+R|}{|R|} = 0$. If $S = \frac{|k+2R|}{|2k+R|}$, find the value of S .

3. 已知 $T = \sin 50^\circ \times (S + \sqrt{3} \times \tan 10^\circ)$ ，求 T 的值。

Given that $T = \sin 50^\circ \times (S + \sqrt{3} \times \tan 10^\circ)$, find the value of T .

4. 已知 x_0 和 y_0 是實數且滿足方程組 $\begin{cases} y = \frac{T}{x} \\ y = |x| + T \end{cases}$ 。若 $W = x_0 + y_0$ ，求 W 的值。

Given that x_0 and y_0 are real numbers satisfying the system of equations $\begin{cases} y = \frac{T}{x} \\ y = |x| + T \end{cases}$. If

$W = x_0 + y_0$, find the value of W .



Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 3 (Individual)

香港數學競賽 (2005 – 2006)

決賽項目 3 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

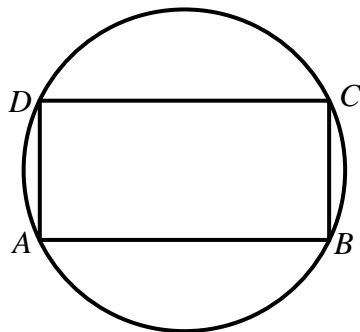
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $\frac{2x-3}{x^2-x} = \frac{A}{x-1} + \frac{B}{x}$ ，其中 A 和 B 是常數。若 $S = A^2 + B^2$ ，求 S 的值。

Given that $\frac{2x-3}{x^2-x} = \frac{A}{x-1} + \frac{B}{x}$ where A and B are constants. If $S = A^2 + B^2$, find the value of S .

2. 如圖一， $ABCD$ 是圓內接長方形， $AB = (S-2)$ cm 及 $AD = (S-4)$ cm。若圓形的圓周是 R cm，求 R 的值。(取 $\pi = 3$)

In Figure 1, $ABCD$ is an inscribed rectangle, $AB = (S-2)$ cm and $AD = (S-4)$ cm. If the circumference of the circle is R cm, find the value of R . (take $\pi = 3$)



圖一

Figure 1

3. 已知整數 x 和滿足 $\frac{R}{2}xy = 21x + 20y - 13$ 。若 $T = xy$ ，求 T 的值。

Given that x and y are integers satisfying the equation $\frac{R}{2}xy = 21x + 20y - 13$. If $T = xy$, find the value of T .

4. 設 a 是方程 $x^2 - 2x - T = 0$ 的一個正根。若 $P = 3 + \frac{T}{2 + \frac{T}{2 + \frac{T}{2 + \frac{T}{a}}}}$ ，求 P 值。

Let a be the positive root of the equation $x^2 - 2x - T = 0$. If $P = 3 + \frac{T}{2 + \frac{T}{2 + \frac{T}{2 + \frac{T}{a}}}}$, find the value of

P .



Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 4 (Individual)

香港數學競賽 (2005 – 2006)

決賽項目 4 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 設 $\frac{k}{4} = \left(1 + \frac{1}{2} + \frac{1}{3}\right) \times \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \times \left(\frac{1}{2} + \frac{1}{3}\right)$ ，求 k 的值。

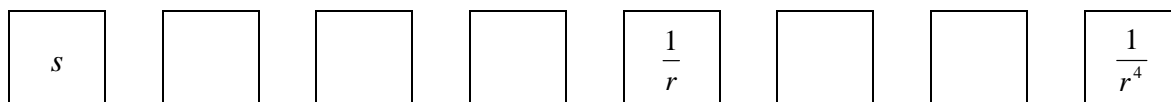
Let $\frac{k}{4} = \left(1 + \frac{1}{2} + \frac{1}{3}\right) \times \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \times \left(\frac{1}{2} + \frac{1}{3}\right)$, find the value of k .

2. 設 x 和 y 是實數且滿足方程 $y^2 + 4y + 4 + \sqrt{x + y + k} = 0$ 。若 $r = |xy|$ ，求 r 的值。

Let x and y be real numbers satisfying the equation $y^2 + 4y + 4 + \sqrt{x + y + k} = 0$. If $r = |xy|$, find the value of r .

3. 如圖一，八個正數排成一列，從第二個數開始，每個數都等於前面兩個數的乘積。已知第五個數是 $\frac{1}{r}$ ，而第八個數是 $\frac{1}{r^4}$ 。若第一個數是 s ，求 s 的值。

In Figure 1, there are eight positive numbers in series. Starting from the 3rd number, each number is the product of the previous two numbers. Given that the 5th number is $\frac{1}{r}$, and the 8th number is $\frac{1}{r^4}$. If the 1st number is s , find the value of s .



4. 設 $[x]$ 表示不大於 x 的最大整數，例如 $[2.5] = 2$ 。若

$$w = 2 + [10 \times s^2] + [10 \times s^4] + [10 \times s^6] + \cdots + [10 \times s^{2n}] + \cdots, \text{ 求 } w \text{ 的值。}$$

Let $[x]$ be the largest integer not greater than x , for example, $[2.5] = 2$. If

$$w = 2 + [10 \times s^2] + [10 \times s^4] + [10 \times s^6] + \cdots + [10 \times s^{2n}] + \cdots, \text{ find the value of } w.$$

